A Methodology for Estimation of Local Heat Fluxes in Steady Laminar Boundary Layer Diffusion Flames

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Abstract

A unique methodology has been used for the estimation of local mass burning rates and flame heat fluxes over a laminar boundary layer diffusion flame with a high accuracy by utilizing micro thermocouple measurements in the gas phase close to the condensed phase surface. Both liquid and solid fuels were investigated. Convective and radiative heat feedback from the flames were measured both in the pyrolysis and plume regions by using temperature gradients near the wall. As expected, for small laminar flames, convective heating was found to be the dominant mode of heat transfer to the condensed fuel surface and accounted for nearly 85-90\% of the total flame heat flux in both liquid and solid fuels. The total average incident flux to the condensed fuel surface was estimated to be approximately 22, 20 and 27 kW/m\(^2\) for methanol, ethanol and Poly Methyl Methacrylate (PMMA) wall-bounded flames, respectively. The average convective heat flux from the flame to the wall in the pyrolysis zone was estimated to be 18.9, 17 and 22.9 kW/m\(^2\) for methanol, ethanol and PMMA, respectively. The radiative component in these small flames was observed to be small, never accounting for more than 20\% of the total wall heat flux. Temperature gradients normal to the wall, proportional to the dominant convective heat flux, were found to decrease from the leading edge towards the trailing edge in the pyrolysis zone, however they were found to remain relatively constant in the combusting plume region (\(\sim 450\) K/mm) until the tip of the flame was reached. Thereafter, they were found to decrease significantly downstream of the combusting plume. The work presented here also discusses the selection of transport properties at

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appropriate temperatures to calculate convective fluxes by using a crude approximation as opposed to detailed temperature measurements in such flames.

*Keywords:* local heat fluxes, local mass burning rates, temperature gradients, boundary layer diffusion flame, vertical wall fire
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<tr>
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<td>$m_{f''}$</td>
<td>Mass-burning rate (mass flux) (kg/m$^2$-s)</td>
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<td>Non-dimensional distance, $y/L$ (-)</td>
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<td>$y_f$</td>
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1. Introduction

In most problems related to fire safety, it is necessary to have a means of estimating the mass-burning rate over condensed fuel surfaces. The rate of flame propagation over any solid or liquid fuel surface, and its ultimate growth to a large fire primarily depends on the burning or mass-loss rate of the fuel and associated heat fluxes to unburnt fuel ahead of the pyrolyzing region. The burning rate also determines flame heights, rates of heat release, and smoke and CO emissions, which in turn determine the growth rate of the fire and its potential hazards. The burning rate and forward flame heat fluxes in turn depend on external conditions, such as the free-stream velocity (for the burning of fuel in a forced convective environment) and the fuel geometry.
such as the angular orientation of the combustible surface with respect to the normal gravity direction. The coupled nature of these properties makes them difficult to determine under mixed conditions.

The work presented here has utilized experimental data of steady laminar flames established over vertical condensed fuel surfaces to separate the convective and radiative components of heat feedback to condensed fuel surfaces. Accurately determining the convective and radiative components of flame heat flux is important for the study of laminar burning fuels and future numerical validation. Although this study only approaches laminar wall fires, smaller in scale than realistic unwanted fires, the laminar wall fire is a canonical fire research problem and serves as an important first step for future development of the techniques and numerical models of fire spread. Detailed high resolution temperature measurements were taken both in the pyrolysis and plume regions of boundary layer diffusion flames using both liquid and solid fuels. Temperature profiles were measured by using fine-wire thermocouples and time-averaged fuel consumption rates were measured using a load cell. The convective heat feedback to the wall was then determined by utilizing the detailed temperature measurements near the wall. Total flame heat flux was measured both in the pyrolysis and plume regions using a standard water-cooled heat flux gauge. By comparing both of these measurements with local mass-loss data from a new technique which utilizes local temperature gradients near the fuel surface [1], convective, radiative and net heat flux components can be extracted locally for this canonical fire research problem. This study therefore seeks to improve the accuracy and predictive capability of theoretical and numerical models while providing an experimental data set for local burning rates and various components of incident flame heat flux to the condensed fuel surface.

2. Literature Review

Burke and Schumann [2] were one of the earliest research teams to present a theoretical analysis of a general diffusion flame from homogeneous reactants. Spalding [3] later addressed the problem of fuel pyrolysis due to energy transfer from a combustion zone. Emmons pioneering work in which he mathematically modeled a steady, laminar diffusion flame in a forced-flow airstream provided a framework for most fire safety studies and laminar diffusion flames [4]. The simplistic closed-form similarity solution presented by Emmons served as a starting point for further studies of steady [5, 6] and
spreading \([7, 8]\) laminar flames. The results of analyses in all configurations showed that the local burning rate per unit area was controlled by the fuel mass-transfer number, \(B\) \([9, 10]\), and that it decreased with distance from the leading edge.

Experimental studies followed analytical work, investigating various aspects of non-spreading, steady, boundary layer type diffusion flames. One of the earliest experimental investigations on the aerodynamic structure and stability of diffusion flames stabilized over a fuel surface was reported by Hirano and co-authors \([11, 12]\), where gaseous fuels were injected uniformly through a porous flat plate into a parallel air stream. In their subsequent experimental studies, Hirano and Kinoshita \([13]\) measured gas velocities and temperature profiles across a diffusion flame established over a liquid fuel surface with a free air stream parallel to the plate. Later, Andreussi and Petarca \([14]\) carried out experiments similar to those by Hirano and Kinoshita \([13]\) using ethyl alcohol as a fuel. Both authors studied the structure of the diffusion flame formed on a liquid surface with a parallel oxidizer flow experimentally, analytically or both. Gas velocity and temperature profiles were measured by Hirano and Kinoshita whereas Andreussi and Petarca measured the temperature and species concentration profiles across the boundary layer diffusion flame. However, both the authors did not measure the local temperature gradients at the surface of a condensed fuel and no attempt was made to measure the local fuel consumption rate. A recent study by the authors \([1]\) utilized a first set of temperature measurements close to the fuel surface of methanol, ethanol and PMMA using a new technique to calculate local mass burning rates.

Several experimental investigations on forced convection boundary layer flames on Poly methyl methacrylate (PMMA) plates have also been reported. Krishnamurthy and Williams \([15]\) investigated forced convective boundary layer flames on a PMMA plate. While there have been several relevant studies on the burning of polymethyl methacrylate (PMMA) slabs \([16, 17]\), only Ananth et al. \([18]\) and later Ndubizu et al. \([19]\) report both average burning rates and temperature profiles for steady and unsteady PMMA burning in a laminar forced convective environment.

Several experimental investigations were reported in the literature to evaluate the contribution of one or more surface heat flux components to the burning rate in case of PMMA slabs. The rate of burning depends on the heat feedback to the condensed fuel surface, and in boundary-layer flames it is highest in the leading section where the flame is close to the surface, while
it decreases with the distance from the leading edge. Orloff and co-authors [20] performed a detailed, careful heat transfer analysis using 1.57-m-high, 0.41-m-wide, and 4.5-cm-thick vertical slabs of transparent PMMA. Burning rate and total outward radiation (flame plus surface) at different heights along the slab were measured. The convective heat flux was inferred from the steady surface energy balance for an infinitely thick slab and subsequently the total heat flux. A slight decrease of the convective flux with height was observed. A little later, Orloff et. al. [21] conducted a similar study on larger samples. Unlike previously, the convective heat flux was assumed to be constant and equal to that measured at midheight, which allowed the calculation of the radiant heat feedback. Beaulieu and co-authors [22] used 9-cm-high black PMMA slabs and quantified the effect of enhanced ambient oxygen concentration on the flame heat flux. They found a total heat flux of 33 kW/m$^2$ using Schmidt-Boelter heat flux gauges that were placed along the centerline above the burning area. Kulkarni and co-authors [23] measured a maximum forward heat flux of 34.6 kW/m$^2$ for a 120 cm $\times$ 30 cm vertical sample of clear PMMA using water-cooled Gardon gauges embedded in the burning area, whereas a value of 31.9 kW/m$^2$ was found for black PMMA slabs. On a larger scale, Wu and Tewarson [24] conducted experiments on 5-m-high, 0.6-m-wide and 2.54-cm-thick vertical PMMA samples. For such large samples, they measured total heat fluxes from the flame to the fuel surface of up to 40 kW/m$^2$. Tsai and Wan [25] estimated the total heat flux to be in the range of 23.5 to 30.2 kW/m$^2$ using 100-cm-high and 2-cm-thick slabs of clear PMMA with widths ranging from 10-70 cm, in relatively good agreement with the values obtained in our study. A more recent work by Pizzo and co-authors [26], in which they studied both the steady and transient burning of thick clear PMMA slabs, suggest total heat flux from the flame to be in the range of 30.9-23.4 kW/m$^2$ as the sample height increases from 2.5 to 20-cm. The values reported for total heat flux seems to be in relatively good agreement with the values obtained in our study.

As stated earlier, the flame to surface heat flux and the length of the region over which this heat flux is imparted to unburnt fuels is usually important in accurate modeling of flame spread. Many studies have estimated this heat flux to the unburnt region with a constant value [27, 28, 23], however most detailed measurements have shown this is only an approximation, and in reality a profile of parabolic or exponentially-decaying heat flux exists [29, 30, 31], especially in smaller flames like those studied here.
3. Local mass burning rate technique

A methodology was developed earlier [1] that allowed for the estimation of local mass burning rates. The method has its basis in the Chilton-Colburn [32] extension to the Reynolds analogy [33] that establishes a relationship between mass, momentum, and heat transfer in a boundary layer over a solid or liquid fuel surface.

\[
\frac{\tau_s}{u_{\infty}\nu^{2/3}} \equiv \frac{h}{c_p\alpha^{2/3}} \equiv \frac{\dot{m}''}{D^{2/3}l\nu(1 + B)}.
\]  

(1)

In his pioneering work, Emmons hypothesized that the shear stress at the surface of a combusting fuel must be proportional to the mass burning rate [4]. Using Eqn. (1), Emmons hypothesis and a heat transfer coefficient at the surface of a flat fuel (outlined in section 5.6), an expression for the local mass burning rate can be derived. By extension of the Reynolds Analogy, it was hypothesized that the non-dimensional temperature gradient at the surface of a condensed fuel is related to the local mass burning rate through some constant of proportionality [1] and was given by

\[
\dot{m}''_f = \frac{C}{L} \left( \frac{\partial T^*}{\partial y^*} \right)_{y^*=0} = \frac{B k_w}{c_p L} \left( \frac{\partial T^*}{\partial y^*} \right)_{y^*=0},
\]  

(2)

where \( T^* = (T - T_{w,p} / T_{fl,ad} - T_{w,p}) \) represents the non dimensional temperature, \( T_{w,p} \) and \( T_{fl,ad} \) represent the wall (taken as the pyrolysis temperature of the given fuel) and adiabatic flame temperature, respectively for a given fuel, \( L \) is a length scale representing the length of the region that is pyrolyzing or vaporizing and \( y^* = (y / L) \) denotes the non-dimensional normal direction with reference to the surface that is issuing fuel vapor. The definition of the non-dimensional temperature was chosen so that the boundary layer equations can be properly normalized. Also, defining \( T^* \) in the manner outlined above helps in making the relationship universally applicable over a wide range of fuels and geometry. The proportionality constant \( C \) appearing in Eqn. (2) equals \( (B k_w / c_p) \), where \( k_w \) is the thermal conductivity of the gas phase evaluated at the wall temperature and \( c_p \) is the mean specific heat measured at the adiabatic flame temperature of the given fuel. The term \( B \) that appears in Eq. (1,2) is a non-dimensional proportionality constant that relates the rate of mass transfer (e.g., vaporization, combustion) to the rate of heat transfer, and is essentially the driving force for mass transfer, and was first referred to as the “transfer number” by Spalding [9]. The Prandtl
number, Pr, is assumed to be equal to unity, which also helps in selecting the transport properties at appropriate temperatures, shown later.

Conceptually, Eqn. (2) states that the proportionality between the velocity gradient at the surface (momentum transfer) and the fuel vaporization rate in a chemically reacting boundary layer will extend to the temperature gradient as well. A similar analysis can be performed to demonstrate the proportionality between the fuel species and mass transfer using the assumption of a constant Schmidt number. For fire science, the utility of Eqn. (2) is evident as it allows estimation of the local mass-burning rate by measurement of the temperature profiles along the fuel surface. The proportionality between the rate of transfer of the fuel vapor and the non-dimensional temperature gradient at the fuel surface is utilized in the subsequent sections in order to estimate net heat fluxes for the pyrolyzing samples, necessary for separation of convective and radiative components of the flame heat flux.

4. Experimental Facility and Instrumentation

An experimental setup was constructed to enable simultaneous measurements of average mass-loss rates and local temperature profiles in a boundary-layer diffusion flame established over a condensed fuel surface. Two U-shaped aluminum brackets were connected to an aluminum sheet and mounted vertically atop a load cell. A sheet of ceramic fiber insulation board 1.27 cm thick, with a section 12.7 cm from the base of the sheet cut out for holding the fuel sample, was mounted atop the aluminum sheet. The front surface of the insulation wall was coated with a black radiation absorbing paint having an absorbtivity of approximately 98%. The liquid fuel wick was an 8 cm × 8 cm × 1.27 cm thick sheet of porous noncombustible material (Alkaline earth silicate wool). In order to eliminate leakage of the liquid fuel from the sides, sodium silicate was applied to all interfaces of the wick except the top face. Burning was limited to the front surface of the wick by shielding the remaining sides with aluminum foil. During testing, the wick was soaked with liquid fuel up to its point of saturation so that it gave a stable boundary-layer diffusion flame for the longest time duration possible (enough to take precise temperature measurements). The fuel wick was soaked with approximately 90 ml of liquid fuel for each test. In case of a solid fuel, an 8 cm × 8 cm × 1.27 cm thick sheet of PMMA was used.

Figure 1 shows a schematic diagram of the experimental setup. The fuel burning rate was measured by monitoring the mass loss of the burning wick
over a timed interval. A Mettler Toledo precision balance was used, which had a maximum capacity of 32.2 kg and a resolution of 0.1 g, to measure the mass-loss rate of the condensed fuel surface. Following ignition, the burning rate (indicated by the rate of mass loss) remained constant for most of the test time. However, as the wick dried up, the burning rate decreased. The burning rate measurements were made in the early stages of the burning process when the average burning rate is constant and governed by the rate of fuel evaporation, rather than diffusion through the fuel wick. The average mass-loss rate of the condensed fuel surface was determined by measuring the slope of the linear mass-loss versus time curve during steady burning. The burning rate measurements presented are averages of six tests at a given condition. The repeatability of these measurements was within 2% of the mean.

Figure 1: Schematic diagram of the experimental setup in a vertical configuration. The black dots above the wick represent the holes drilled for wall heat flux measurements by a water-cooled heat flux gauge.

Precise temperature measurements were carried out using R-type Pt/Pt-13% Rh micro thermocouples (spot welded) of 50 μm (0.002 in) and 75 μm (0.003 in) wire diameter with a bead of approximately 100 μm and 150 μm in diameter respectively (according to manufacturer’s specifications). In prac-
tice, most thermocouples have diameters in the range $1.5 \, d_w < d_b < 2.5 \, d_w$ (in our case $d_b \sim 2d_w$). The micro thermocouple wires were housed in a single 1.1 mm diameter twin bore ceramic cylinder and a smaller length of thermocouple wire was exposed ($\sim 10 \text{ mm}$). The exposed wire was shaped into a flat semi-circular form. Probe-induced perturbations are reduced in this single support design. Figure 2 shows a schematic of the given thermocouple single support design. Both 50 $\mu m$ and 75 $\mu m$ wire-diameter thermocouples were used over the same sample in order to ensure accurate radiation corrections by reading the difference between these two at the same location and applying the correlation of Collis and Williams [34]. For the 50 $\mu m$ wire-diameter thermocouple, a typical radiation correction at 1700 K was found to be approximately +83 K. Since the thermocouples cross regions of high temperature gradients, the measurements are expected to include conduction errors, however they have been estimated to be small here ($<1\%$) since the heat-transfer area (the cross section of the thermocouple) is very small, therefore no corrections were made in the data for conduction errors. More details on neglecting a conduction correction for the given thermocouple can be found in the supplementary material.

Figure 2: Schematic of thermocouple single support design.

Micro-thermocouples were mounted to a set of computer-controlled Velmex X-Y unislides such that they can be moved precisely up and down along the flame length or left and right across the flame thickness with a maximum spatial resolution of 1.5 $\mu m$. Voltage signals from the thermocouples were acquired, conditioned and digitized through a National Instruments NI 9214, which is a 24-bit high density 16-channel thermocouple input module and
can be used up to 0.02°C measurement sensitivity. The LabVIEW software was used for synchronized motor control and continuous temperature data acquisition. Because the temperature gradients are very steep within a boundary-layer diffusion flame, very fine movements were necessary. Through numerical testing, it was determined at least 22-36 measurements at 0.5 mm intervals should be made across the flame in order to fully resolve the thermal boundary layer. Measurements were taken at 0.25 mm spacings near the condensed fuel surface and then subsequently incrementing the spatial distance to 0.5 mm. The thermal boundary layer thickness for a methanol flame is approximately 18 mm at the trailing edge of the sample surface. Figure 3 shows the photograph of the experimental set-up.

In the case of a PMMA slab, steady burning samples were ignited by a standard blowtorch, which was passed over the surface for approximately 60 s until the entire surface was uniformly ignited. Thus, a boundary-layer flame was established over the entire surface and every point on the surface was exposed to the flame for the entire duration of the test. Experimental time was started immediately after uniform ignition. Unlike liquid wicks, as the PMMA sample burns its surface regresses with time, therefore it was necessary to complete the temperature mapping while the surface remained
relatively flat, measured to be within approximately 150 s following uniform ignition. 5-point temperature measurements from the molten layer into the gas-phase at 0.25 intervals were recorded at 12 $x$-locations within 150 s of ignition. PMMA samples were also burned for different time intervals to assess errors from surface regression and to calculate the local mass-burning rates for comparison. The sample was allowed to burn for a known time interval before the flame was extinguished. Since the PMMA surface regresses with time, it becomes difficult to measure the molten layer temperature at the PMMA surface. However, it becomes imperative and essential to measure the molten layer temperature in order to compute temperature gradients accurately. Therefore, measurements with one thermocouple at various stream-wise locations was made by carefully moving the thermocouple down through the flame until it penetrated the molten layer.

The heat flux to the wall in the overfire region was measured using a total heat flux gauge. Holes were drilled into the insulation board at several locations in the overfire region for using a water-cooled Medtherm total heat flux gauge (model 64-10-20). For measurements with the heat flux gauge, the data acquisition system acquired samples at a sampling rate of 1000 Hz for a total time duration of 500 s. The data was then processed and averaged over the time duration of steady burning of liquid and solid fuels. In all tests, total flame heat flux was measured at 14 locations downstream of the condensed fuel surface. Temperature measurements were also carried out in the plume region at several locations downstream of the pyrolysis region. Temperature measurements were carried out at 7 stream-wise locations (at every other measurement location), downstream of the pyrolysis zone. Detailed temperature profiles in the combusting and thermal plume ensured calculation of convective heat fluxes with high accuracy.

Measurements of the flame stand-off distances were recorded by digital photographs, where the distance from the condensed-fuel surface to the center of the blue flame zone was measured and taken as the position of the flame. The flames were photographed in a darkened room with a side-view digital camera (Canon EOS). Before a sample was ignited, the camera was calibrated by taking a picture of a sheet of graph paper that was aligned along the vertical axis of the fuel surface. Four calibration images were taken. An average pixel/mm count was then obtained from the four images taken. This value of pixels/mm was later used during the postprocessing of the images while calculating the standoff distance. The field of view was chosen to reduce errors in the stand-off distance measurement to less than 5%. A field
of view at the center of the fuel specimen covering an area of 20 cm × 14 cm was imaged for calculating the flame stand-off distance in the pyrolysis zone. The digital images were averaged in Matlab and flame stand-off distances were measured by using ImageJ software. In a particular test, 200 images were averaged during the steady burning regime time to obtain an averaged image. Flame stand-off measurements were carried out independently for 3 repeated tests at a given flow condition. The results were then averaged to give an averaged flame stand-off distance profile. The repeatability of these measurements was within 2% of the mean. Figure 4 shows a larger picture of the flame with a marker representing the center of the blue flame zone.

![Figure 4: Photograph of a free-convection methanol diffusion flame in color, grayscale and binary mode. The red marker represents the center of the blue flame zone.](image)

During the experimental tests, the data acquisition system acquired temperatures at 100 samples per second, providing 100 samples to average per spatial point. The dwell time at each point was 1 s. Reported temperatures are averages of at least five tests in a given condition and the maximum standard deviation was < 3.2% of the mean for the pyrolysis region and < 6% of the mean for the plume region. The inherent uncertainty in temperatures measured by the thermocouple \( T_{t_c} \) are taken to be 0.25% of the measured value based on manufacturers specifications. The accuracy of the Nusselt
number correlation used to calculate the radiation loss from the thermocouple bead was reported to be within 5% \cite{34} and the uncertainty in \(k\) due to different species is assumed to be 3%. The error in the thermocouple emissivity used (\(\varepsilon_{tc}\)) is also small, \(< \pm 3\%\), except that \(\varepsilon_{tc}\) is linear with \(T_{tc}\) so any error in \(T_{tc}\) increases the uncertainty in \(\varepsilon_{tc}\). The Platinum emissivity was calculated using Jakobs theoretical correlation, confirmed by experimental data \cite{27-28} which reported the Pt emissivity uncertainty \(< \pm 3\%\) when using the calculation. The uncertainty in gas temperature is then calculated from a quadratic sum of the uncertainties:

\[
dT_g = \left[ \left( \frac{\partial T_g}{\partial T_{tc}} dT_{tc} \right)^2 + \left( \frac{\partial T_g}{\partial \varepsilon_{tc}} d\varepsilon_{tc} \right)^2 + \left( \frac{\partial T_g}{\partial k} dk \right)^2 + \left( \frac{\partial T_g}{\partial Nu} dNu \right)^2 \right]^{1/2}.
\]

The maximum uncertainty in gas temperatures encountered in the flame zone is then found to be within \(\pm 10\) K.

5. Results and Discussions

5.1. Gas-phase temperatures

Using the experimental apparatus described in Section 4, measurements were taken for average mass-burning rates and local temperature profiles along condensed fuel surfaces. Figure 5 shows the temperature profiles at several stream-wise locations along the condensed fuel surface for a methanol, ethanol and PMMA boundary-layer diffusion flame. Under a free-convection environment, the temperature increases in the perpendicular direction to a maximum value and then decreases to an ambient temperature at some location away from the surface. Observation of the temperature gradients normal to the fuel surface suggests that they are highest near the leading edge and decrease further downstream. The peak temperatures at the flame front show a similar trend, reaching a maximum of 1915 K and 1888 K for the methanol and ethanol flames, respectively, \(\sim 10\) mm downstream of the leading edge. At the trailing edge of the sample, the peak temperature drops by about 151 K and 173 K for methanol and ethanol wall flames, respectively. This temperature decrease is primarily due to convective heat losses. The temperature of the condensed fuel surface also decreases with \(x\), however this change was very slight. The temperature of the condensed fuel surface was found to be approximately near the boiling points of the fuels studied here.
Figure 5: (left) Temperature profiles at several locations along the fuel surface of length 8 cm for a methanol, ethanol and PMMA boundary layer diffusion flame. (right) Variation of the PMMA molten layer temperature along the length of fuel surface.

Figure 5 also shows the temperature profiles at several stream-wise locations for a PMMA slab. The maximum peak temperature for PMMA is about 1923 K, which occurs about \( \sim 10 \) mm from the leading edge. Thereafter, the value of the peak temperature decreases slowly, signaling the end of heat-release region. It dropped by about 198 K close to the sample trailing edge. Molten layer temperatures at various stream-wise locations were also measured for a PMMA slab, shown in Figure 5 as a function of \( x \). The temperature of the molten layer decreased slowly with \( x \), moving downstream of the leading edge. The average temperature of the molten layer was found to be 677 K. The convective heat feedback to the surface was also found to decrease with \( x \). In the current tests, the convective heat flux has been estimated to be several times higher at the leading edge than in the trailing section, 26.1 kW/m\(^2\) at \( x = 10 \) mm and 14.0 kW/m\(^2\) at \( x = 80 \) mm for a methanol flame and 32.4 kW/m\(^2\) at \( x = 10 \) mm and 15.0 kW/m\(^2\) \( x = 80 \) mm for a PMMA flame. The convective heat flux was evaluated here by measuring temperature gradients at the condensed fuel surface and using the expression \( k_w (\partial T / \partial y) y=0 \). Thus, both the heat feedback and the measured pyrolysis temperature decreased with \( x \), but the pyrolysis temperature decreased more slowly. This is consistent with the results of Vovelle et al. [35], who reported that the pyrolysis temperature (in the case of PMMA) increases slightly with the increasing heat flux to the surface. Also, wall temperature decrease more rapidly with \( x \) for PMMA when compared to
methanol and ethanol fuel surface temperatures. The local mass-loss rate from the fuel, driven by convective heat fluxes to the surface in these small, laminar flames, should similarly decrease with $x$. A comparison of the shapes of the temperature profiles at various $x$ locations reveal that the temperature gradients decrease with $x$ from the leading edge. This is consistent with known characteristics of boundary-layer diffusion flames, in that convective heat feedback decreases with $x$. The flame usually becomes thicker when moving downstream of the leading edge and the flame stand-off distance increases with increasing $x$. It follows, therefore, that the local mass burning-rate should also decrease with $x$, discussed later.

5.2. Flame stand-off distance

The non-dimensional flame stand-off distance $y_f^* = (y_f/L)$ at different locations along the fuel surface are plotted in Figure 6 for both methanol and ethanol boundary-layer diffusion flames. The experimental flame stand-off distance is taken as the distance measured in the normal direction to the fuel surface to the point where the highest temperature was recorded. The flame stand-off distance measured through photographs are also plotted against the experimental results. Reasonably close agreement of the flame stand-off distance measured by both methods confirms the accuracy of measurements. The flame stand-off distance is lower near the leading edge and it increases further downstream up to the trailing edge. This leads to enhanced heat transfer to the fuel surface at the leading edge and hence higher evaporation rates of the fuel at this location. Accordingly, the local burning rate is highest at this location. The flame stand-off distance is higher for the regions near the trailing edge and hence heat transfer at these locations is lower. Due to lower heat transfer rates at these locations, the local mass-burning rates are found to be lower at these locations. Also, the non-dimensional flame stand-off distance in both the methanol and ethanol cases is almost proportional to $x^{0.25}$, confirming the similarity theory for a natural-convection boundary layer adjacent to a vertical flat plate [6, 7]. However, for PMMA the non-dimensional flame stand-off distance does not follow the boundary layer similarity theory and was found to be proportional to $x^{0.34}$. It will later be shown that the PMMA local pyrolysis rate does not follow the similarity theory either.
5.3. PMMA regression and local pyrolysis rate

The PMMA sample is allowed to burn for a known time interval before the flame is extinguished. After the sample cools, the sample is cut along the center-line and its thickness along the center-line is measured at various $x$ locations. The results presented in this section are the average of three independent tests performed under the same atmospheric conditions. The sample regression at each location was obtained as the difference in thickness (before and after the test) divided by the test duration. It is to be noted that the regression rate calculated by this method is a time-averaged regression rate and may therefore be different from the more desirable instantaneous
regression rate. PMMA thermal expansion does not cause any significant variation in the results as they are measured after the sample cools, therefore, the measured thickness was not corrected for thermal expansion. Figure 7 (a) shows the regression profile of the PMMA surface at different burn-out times. Significant regression was observed in the leading section when compared with regression at locations downstream. This was found to be consistent with the non-uniform rate of heat feedback from the boundary-layer flame to the PMMA surface. At higher burn-out times \( t > 250 \) s, the sample surface receded significantly below the level of the original surface. This resulted in the formation of a valley or step in the leading section as shown in Figure 7 (a). Figure 7 (a) demonstrates that the valley deepens and the position of the deepest point moves slightly downstream with time.

Figure 7 (b) present the time-averaged regression rate at various stream-wise locations. It is observed that the burned thickness is much larger up-stream than downstream and similarly larger in the 850 second duration test than in 150 s test. Therefore, the error in burning rate measurement should be highest downstream at short times. Figure 7 (b) shows that the regression rate peaks near the leading edge and thereafter decreases sharply with \( x \) as we move downstream of the leading edge, consistent with the way heat feedback varies with \( x \). The rate of decrease with \( x \) is slower at longer burn times. The rate also decreases sharply from the peak towards \( x = 0 \), especially at burn times of 550 s and 850 s. A sudden increase in the regression rate was observed at the trailing edge \((x= 80 \text{ mm})\) which is primarily due to edge effects. It was observed that the surface was still flat near the trailing edge for a burn-out time of 250 s and becomes increasingly curved for longer burn times. This non-uniform regression leads to the formation of a valley that deepens with time, which affects the local burning rate. Figure 7 (b) also shows that the surface regression rate decreased with time in the leading 2 mm of the sample. Also, the peak regression rate decreased from 0.02175 to 0.00910 mm/s (a factor of 2.39) between the 100 s and 850 s tests. The \( x \)-position of the peak regression rate was observed to shift slightly downstream with time. It is clearly seen from Figures 7 (a) and (b) that uneven surface regression and the creation of a valley led to the transient surface regression rate in the leading section, supporting the observations made by Ndubizu et al. [19]. The heat feedback to the sample surface decreases significantly in the mid-section of the sample and therefore the degree of surface regression is small.

The local burning or pyrolysis rate is time dependent and this can be
clearly seen in Figure 7 (b). Closely observing the regression rate profiles for \(40 \leq x \leq 70\) mm (neglecting the data at \(x = 80\) mm due to edge effects), it is observed that the regression rate for tests at higher burn-out times is distinctly the largest at each \(x\) location, and a closer look at the data reveals an increasing trend. This observation is similar to the observations made by Ndubizu et al. [19]. The regression rate definitely increases with time in the trailing section and this is probably due to increased heat feedback with time and slow in-depth heat conduction in this section, where the heat feedback rate is small relative to that in the leading section.

![Figure 7: (a) (left) Regression profiles of the PMMA surface at different burn-out times (b) (right) PMMA surface regression rate at various stream-wise locations in tests lasting 100, 150, 250, 550 and 850 seconds.](image)

5.4. Non-dimensional Temperature Gradients and Local Mass Burning Rate

Figure 8 shows the variation of the normal non-dimensional temperature gradient along the fuel surface extracted from experimental temperature data of methanol, ethanol and PMMA. The normal non-dimensional temperature gradients at the fuel surface, \(\left. (\partial T^*/\partial y^*) \right|_{y^*=0}\) were calculated from the slope at \(y^*=0\) of a third order polynomial fit to the non-dimensional temperature distribution near the fuel surface. The temperature gradient normal to the fuel surface was found to be highest at the leading edge and lowest at the trailing edge (\(x = 80\) mm). The local mass-burning rate should follow a similar trend, as is revealed by the calculated rates in Figure 9.

Averaging the non-dimensional temperature gradient for the entire fuel surface from data in Figure 8, the average mass-burning rate is estimated to
be 18.0 g/m²s, 20.2 g/m²s and 9.8 g/m²s for methanol, ethanol and PMMA respectively, using Eq. (2). Appropriate average values of transport properties were used to calculate the constant $C$ in Eq. (2) and are given in Table 1. It is to be noted that we evaluated the transport properties following the work of Kim et al. [6] where they evaluated the value of $k$ at the wall and the value of $c_p$ at the adiabatic flame temperature of the given fuel. The assumption of a unity Prandtl number in deriving Eqn. (2) also supports such selection of transport properties, as evaluation of the transport properties at the wall, namely $\mu_w$ and $k_w$, and evaluation of the specific heat at the adiabatic flame temperature of the given fuel results in a unity Prandtl number. Choosing the transport properties as outlined above works well in estimating the average mass burning rates for both liquid and solid fuels. The mass-burning rate evaluated though the load cell data was found to be 16.7 g/m²s, 17.6 g/m²s and 8.9 g/m²s for methanol, ethanol and PMMA respectively. The error in the estimation of the average mass burning rate was therefore found to be $+7.8\%$, $+14.8\%$ and $+10.1\%$ for methanol, ethanol and PMMA respectively. Factors causing the theoretical relationship from Eqn. (2) to over-estimate the average mass burning rate might include assumptions of a unity Lewis and Prandtl numbers, a lack of radiation effects and uncertainties associated with the calculation of transport properties.

Table 1: Physical properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Methanol</th>
<th>Ethanol</th>
<th>PMMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity, $k_w$ (W/m-K)</td>
<td>0.028 [37]</td>
<td>0.029 [37]</td>
<td>0.050 [37]</td>
</tr>
<tr>
<td>Specific heat, $c_p$ (J/kg-K) evaluated at the adiabatic flame temperature</td>
<td>1394.5 [37]</td>
<td>1417 [37]</td>
<td>1658.5 [37]</td>
</tr>
<tr>
<td>$T_{w,p}$ (K)</td>
<td>337 [36]</td>
<td>351 [36]</td>
<td>668 [38]</td>
</tr>
<tr>
<td>$T_{fl,ad}$ (K)</td>
<td>2150 [37]</td>
<td>2195 [37]</td>
<td>2494 [37]</td>
</tr>
<tr>
<td>Length of the condensed fuel surface (m)</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Recall that the temperature measurements for liquid fuels were taken in a steady burning regime, determined by global mass-loss data, however for PMMA the measurements were carried out within the first 150 s (after the surface is uniformly ignited, while the surface was still relatively flat), therefore the mass-loss rate reported is an average for this 150 s region. In the PMMA case, even though solid fuel burning does not yet reach a steady state, the gas-phase reaches a steady state soon after uniform ignition. At the leading edge of the solid, where heat feedback is high, a local gas-phase steady state is achieved soon after ignition, before a significant valley is formed. In that period, approximately all the heat feedback is used to gasify the solid. Later, a valley is formed and the surface regression rate decreases with time in the leading section (outlined in Section 5.3). This suggests that the heat reaching the surface in this section is no longer steady at times later. The same observations were made by Ndubizu et al. [19] when they burnt PMMA slabs in forced convective environments. Ndubizu and co-authors also observed that a more efficient mixing of oxygen and fuel was obtained near the leading edge of the sample at earlier stages of burning while the sample was still flat. However, as the test progressed and the valley formed and deepened, it resulted in a richer fuel mixture in the leading section of the sample. They speculated that because of the valley, the combustion products in the valley were not swept away as readily as was the case when the surface was flat. They suggested that the oxygen mass fraction within
the valley would decrease as the valley deepens. This would result in a drop in the flame temperature and, consequently, a drop in total heat feedback to the surface. They also found that the temperature gradients close to the PMMA surface decreased with time. The same observations were made in this study. After ignition, it was visually observed that the flame near the leading edge was bluish while the leading section of the sample was still flat. This is an indication that a more efficient mixing of oxygen and fuel was obtained in this region of the flame at this stage. However, as the test progressed and the valley formed and deepened, the length of this blue portion shrank, suggesting that the moving boundary layer effects resulted in a richer fuel mixture in this section confirming the observations of Ndubizu et. al. [19]. Similarly, temperature gradients normal to the PMMA surface were found to decrease with time as the valley formed and deepened. No change in the temperature gradients was observed during the initial stages of burning while the surface was relatively flat. This strongly suggest that the heat feedback to the curved sample surface decreased with time as the valley deepens. Therefore, it becomes essential to accomplish steady mass loss rate and temperature measurements during initial stages of burning while the surface is relatively flat. It is to be noted that for PMMA, both the temperature and mass loss rate measurements were carried out within the first 150 s (after the surface is uniformly ignited, while the surface was still relatively flat).

Figure 9 shows the variation of the local mass-burning rate for vertically-oriented methanol and ethanol flames, using the theoretical correlation from Eq. (2) and the non-dimensional temperature gradients at the condensed fuel surface. Due to the availability of fresh oxidizer, higher convective heat feedback, higher temperature gradients and lower stand-off distances near the leading edge, the local burning rate is highest here and subsequently decreases as we move downstream towards the trailing edge. The burning rate decreases, due to the lack of fresh oxidizer, lower convective heat feedback, lower temperature gradients and higher flame stand-off distances as we move downstream. Also, the local mass-burning rate for both methanol and ethanol is almost proportional to $x^{-0.25}$, confirming the power-law relationship for laminar natural convective burning on a vertical surface [5, 6]. The local mass-burning rate evaluated by using Eq. (2) was also compared against the theoretical mass-burning rate given by Kim, de Ris and Kroesser [6]. For laminar natural convection burning, the mass-burning rate on a vertical surface was found to be [6].
\[
\dot{m}_f'' = 3 \left[ \frac{\rho_w^2 \mu_w^2 L_v g \cos \phi}{4 \bar{c}_p T_\infty x} \right]^{1/4} [-f(0)],
\]

where \( \rho_w \) represents the density of the gas phase at the wall, \( \mu_w \) dynamic viscosity at the wall, \( L_v \) the effective heat of vaporization, \( g \) acceleration due to gravity, \( \phi \) the angle from vertical, \( \bar{c}_p \) the mean specific heat for free convection, \( T_\infty \) the ambient temperature and \( x \) the coordinate parallel to the fuel surface. The transport properties were evaluated at the wall temperature, \( T_w \). \([-f(0)]\) was numerically calculated for various liquid fuels elsewhere [6]. The close agreement in the theoretical and experimental local mass burning rates obtained by using the theoretical formulation in Eq. (4) and Eq. (2) suggest that the proposed correlation (Eq. 2) works well in estimating the local mass burning rates for the liquid fuels.

Unlike liquid fuels, for PMMA the local mass-burning rate can be approximated a posteriori by measuring the local surface regression over fixed intervals of time [39]. Once steady burning has been established the pyrolysis mass flow rate for PMMA is computed at each \( x \) location along the central symmetry axis using the first-order approximation given by Pizzo et al. [39],

\[
\dot{m}'(x,t) = \rho_s \frac{d\delta}{dt} \approx \rho_s \frac{\delta(x,t+\Delta t) - \delta(x,t)}{\Delta t}.
\]

Using a PMMA density \( (\rho_s) \) of 1190 kg/m\(^3\) and measuring regression from samples burned for fixed 50 s time intervals after ignition the local mass-burning rate over time can be assessed. A smaller time step would be desirable; however errors in measuring regression profiles become too apparent when the time step is less than 50 s. The regression data between burnout times of 100 and 150 s were used to evaluate the local mass-burning rate in Figure 9 for PMMA. Both the regression-measured and Eqn. (2) calculated mass-burning rates follow a power law decay in \( x \) with exponent -0.35 and -0.31, respectively. The same power law relationship for local mass-burning rates was observed by Pizzo et al. [39] for vertical PMMA slabs of length 2.5 to 20 cm. The departure from the \( x^{-0.25} \) dependence predicted by the LBL theory could be due to a combination of the following reasons (1) the variations of flame and surface temperatures with \( x \) preclude the self-similar solution of boundary layer, and (2) radiative heat feedback to the solid does not follow the boundary layer scaling rule as discussed later. Jiang et al. [40] also observed that the downstream flame, in the case of PMMA, deviated
from the self-similar boundary layer scaling relation. They observed that the variation in flame stand-off distance and the fuel vapor blowing rate with \( x \) does not follow the \( x^{-0.25} \) dependence, as implied by the self-similar solution of the boundary layer. The close agreement, however, in estimating the local mass-burning rates by both the regression data and temperature gradients by using Eqn. (2) suggest that the proposed theoretical correlation works well for both liquid and non-charring solid fuels, as the experimental mass burning rate was not used in any way to obtain the theoretical mass-burning rate.

Figure 9: Variation of the local mass burning rates at the methanol/ethanol condensed fuel surface (left) and PMMA surface (right) along its length.

5.5. Wall Heat Fluxes in the Pyrolysis Zone

Utilizing gas-phase temperature measurements and local mass-burning rates, heat fluxes were evaluated in the pyrolysis zone at various stream-wise locations along the condensed fuel surface. Reasonable approximations were made to simplify the heat balance analysis. The fuel surface was assumed to be opaque with an emissivity and absorptivity of unity. The surface radiative heat loss was given with respect to \( T_\infty \). The energy balance at the condensed fuel surface \( (y = 0) \) for steady burning of liquid fuels becomes

\[
\dot{m}_f'' L_v = \dot{q}_{\text{f,l},c}'' + \dot{q}_{\text{f,l},r}'' - \dot{q}_{\text{s,rr}}'' \tag{6}
\]

and

\[
\dot{m}_f'' L_v = k_w \left( \frac{\partial T}{\partial y} \right)_{y=0} + \dot{q}_{\text{f,l},r}'' - \sigma \left( T_w^4 - T_\infty^4 \right), \tag{7}
\]
where $\dot{q}_{fl,c}'', \dot{q}_{fl,r}'', \dot{q}_{s,rr}'$, and $L_v$ represents the convective heat flux, radiative heat flux, re-radiation heat flux from the surface and effective heat of vaporization or gasification, respectively. Here, the convective heat flux is measured by using the expression $k_w \left( \partial T/\partial y \right)_{y=0}$. The flame imparts heat feedback to the condensed fuel surface primarily in two modes: convective and radiative. For steady burning, the convective heat flux from the flame is equal to $k_w \left( \partial T/\partial y \right)_{y=0}$ at the condensed fuel surface due to the stagnant film hypothesis. Therefore, the convective heat flux from the flame is equal to the conductive heat flux at the fuel surface for steady burning.

For a transparent polymer, here clear cast PMMA, the local mass-loss rate due to pyrolysis, $\dot{m}_f''$, can be related to heat fluxes through an energy balance equation at the surface of the burning material as,

$$\dot{m}_f'' h_g = \dot{q}_{fl,c}'' + (1 - r) \dot{q}_{fl,r}'' + \dot{m}_f'' h_{ng} - \dot{q}_{s,rr}'' - \dot{q}_{id,cond}'' - \dot{q}_{id,r}''. \quad (8)$$

The left-hand term represents the heat carried by fuel vapors away from the surface. The right-hand terms represent heat fluxes from the flame (convective and radiative components), heat carried to the surface by the polymer in its non-volatile state, the outward surface re-radiation and the in-depth heat losses into the solid (namely conduction and radiation), where $r$ represents the surface reflectivity. The term $h_g$ is the effective heat of gasification commonly represented as the effective heat of vaporization. In the case of PMMA, the energy required to raise the temperature of a unit mass of polymer from ambient temperature to the final pyrolysis temperature ($T_p$) including melting, is given by $h_{ng} = h_m + \int_{T_{\infty}}^{T_p} c_p(T)dT$, where $h_m$ is the heat of melting. Equation (8) slightly differs from that given by Orloff et al. [20], who disregarded melting, assuming that the polymer sublimes at a critical temperature and remains inert at temperature below this. As stated by Orloff et al. [20, 21] for steady burning of an infinitely thick slab, the heat carried to the surface by the solid exactly equals the conduction plus net radiation into the solid, yielding

$$\dot{m}_f'' h_{ng} = \dot{q}_{id,cond}'' + \dot{q}_{id,r}''. \quad (9)$$

Following Orloff’s hypothesis and considering surface reflectivity to be zero, Equation (8) can be reduced to

$$\dot{m}_f'' h_g = \dot{q}_{fl,c}'' + \dot{q}_{fl,r}'' - \dot{q}_{s,rr}''. \quad (10)$$

26
which is of the same form as represented by Eqn. (6). This indicates that the calculation of the steady burning rate does not require the estimation of heat losses into the solid interior.

For a boundary-layer diffusion flame, the convective heat flux can be further approximated as [36],

$$
\dot{q}_{fl,c}'' \approx h(T_{fl} - T_w) \approx \frac{k_f(T_{fl} - T_w)}{y_f}
$$

(11)

where

$$
h \approx \frac{k_f}{y_f}.
$$

(12)

This crude approximation allows us to calculate the convective flux in boundary-layer diffusion flames by estimating the wall and flame temperatures at various stream-wise locations along the pyrolysis zone together with knowledge of flame stand-off distances. $k_f$ in the above equation is the thermal conductivity of the gas phase evaluated at a mean film temperature (preferably mean of the actual flame and wall temperatures). In order to calculate convective fluxes by using Eq. (11) above, it is very important that flame and wall temperatures must be accurately determined along with precise measurements of flame stand-off distances. Errors in estimating $k_f$, $y_f$, $T_{fl}$ and $T_w$ could lead to serious deviations in estimating convective heat fluxes by using Eq. (11) above. However, using temperature gradients at the fuel surface is the most accurate way to evaluate the convective heat flux and will be compared to this crude approximation.

$\dot{q}_{s,rr}''$ can be evaluated by knowledge of the wall and ambient temperatures, respectively. Utilizing the theoretical correlation in Eqn. (2), the net heat flux, $\dot{q}_{net}''$ ($\dot{q}_{net}''=\dot{m}'_f L_v$), can be estimated at various stream-wise locations along the pyrolysis zone simply by the knowledge of local mass burning rates along the condensed fuel surface. The effective heat of gasification or vaporization was taken to be 1.2, 0.97 and 1.63 kJ/g for methanol, ethanol and PMMA, respectively [36]. $\dot{q}_{fl,r}''$ can then easily be computed by using Eqn. (7) above. The total heat flux incident to the surface, $\dot{q}_{s,i}''$, can be defined as the sum of the convective and radiative components of the flame heat flux. Figures 10 and 11 show the various components of flame heat flux in the pyrolysis zone of a methanol, ethanol and PMMA vertical wall flame, respectively.

Based on these results, the convective heat flux is relatively high and contributes approximately 85-90% of the total heat flux. Thus, convection
is the dominant mode of heat transfer and radiative contributions are small. This is reasonable for the small, laminar flames studied here. The radiant component could increase downstream where the emissivity may increase if the soot volume fraction is high. The net heat feedback to the condensed fuel surface is the sum of the convective and radiative components minus reradiation from the surface.

For methanol and ethanol flames, all 3 components, namely $\dot{q}_{s,i}'', \dot{q}_{\text{net}}''$ and $\dot{q}_{\text{fl,c}}''$ follow the power law decay and were found to be almost proportional to $x^{-1/4}$, which in turn seems to follow the boundary-layer scaling rule. However, radiative heat feedback to the condensed fuel surface does not follow the boundary layer scaling rule. The re-radiation flux from the condensed fuel surfaces was found to be negligible for both methanol and ethanol diffusion flames. With the knowledge of local distribution of various components of heat flux one can further compute the average value of the given components by using

$$\dot{q}_{\text{avg}}'' = \left( \frac{1}{L} \right) \int_0^L \dot{q}'' dx.$$  \hspace{1cm} (13)

The total incident heat flux from the flame to the wall was estimated to be 21.9 and 19.9 kW/m$^2$ respectively for methanol and ethanol. The average convective heat flux from the flame to the wall was estimated to be 18.9 and 17 kW/m$^2$, for methanol and ethanol, respectively. The average radiative heat feedback from the flame was then calculated to be 3 and 2.9 kW/m$^2$, respectively for methanol and ethanol. Therefore, the radiative heat flux in the pyrolysis zone is only 13.7% and 14.7% of the total wall heat flux in case of methanol and ethanol, respectively. Thus, convection is the dominant mode of heat transfer in steady laminar boundary-layer diffusion flames and is primarily responsible for the pyrolysis of the fuel.
For PMMA, all 3 components of heat flux, namely $\dot{q}_i''$, $\dot{q}_{net}''$ and $\dot{q}_c''$ follow a power-law decay in $x$. The total incident flux on the surface was found to follow the laminar boundary-layer (LBL) scaling rule, however, the net and convective heat flux do not follow the LBL scaling rule. Since the net heat feedback to the solid does not follow this scaling, it follows that pyrolysis or the mass burning rate will not follow the LBL scaling rule either as outlined in section 5.4. Radiative heat feedback to the solid was found to increase towards the trailing edge, most likely due to an increased volume of soot fraction at the trailing edge.

Re-radiation from the PMMA surface was found to be significant when compared to liquid fuels. This is primarily due to increased surface temperatures in the case of PMMA. The surface temperature for PMMA was found to be higher than in the case of liquid fuels (approximately twice that of liquid fuels). Recall that the average wall temperature for PMMA was measured to be 677 K. The total average incident heat flux to the wall was estimated to be 27.0 kW/m$^2$. The average convective heat feedback from the flame to the wall was estimated to be 22.9 kW/m$^2$. The average radiative heat flux from the flame to the wall was then calculated to be 4.1 kW/m$^2$. Therefore, the radiative heat flux is 15.3% of the total incident heat flux. The radiative heat flux component increases for solid PMMA because the flame is more sooty when compared to methanol and ethanol. However, the radiative component still does not exceed 20% of the total incident flux. This is because small laminar flames do not radiate out a significant portion
of the heat released and because the convective heat flux is the dominant mode of heat transfer in such small flames. The average net heat flux that actually pyrolyzes the solid PMMA was estimated to be 16 kW/m². It is to be noted that in the case of PMMA, the average net heat flux which is basically the sum of the convective and radiative heat fluxes minus reradiation from the surface, was found to be quite lower than the total incident heat flux. This can be explained by the fact that a portion of the total incident flux is utilized in pyrolyzing the solid PMMA, whereas the rest is lost into the ambient surroundings through re-radiation from the solid wall. PMMA loses a considerable portion of total heat received by re-radiation compared to liquid fuels, owing to its higher surface temperatures.

Figure 11: Distribution of various components of heat flux in the pyrolysis zone for a PMMA diffusion flame.
5.6. Convective Heat Transfer Coefficient

Convective heat transfer coefficients were also calculated for methanol, ethanol and PMMA wall flames. For a boundary-layer diffusion flame established over a condensed fuel surface, convective heat transfer coefficients can be derived from the expression [1] [41],

$$h = \frac{k_w (\partial T/\partial y)_{y=0}}{T_{fl,ad} - T_{w,p}} = + \frac{k_w}{L} \left( \frac{\partial T^*}{\partial y^*} \right)_{y^*=0}. \quad (14)$$

The above expression comes from normalizing the boundary-layer equations by defining dimensionless independent variables [41]. Figure 12 shows the distribution of the convective heat transfer coefficient for a methanol, ethanol and PMMA wall flame, respectively. The theoretical results based on a thermal boundary-layer approximation [42] and using this crude approximation from Eqn.(12) are plotted for comparison. For both methanol and ethanol, the convective heat transfer evaluated through experiments follows a power-law decay in $x$ consistent with the boundary-layer scaling rule. Experimentally-determined heat transfer coefficients were found to be very similar to those predicted by theory for both the methanol and ethanol boundary-layer diffusion flames. However, for PMMA the heat transfer coefficients determined experimentally diverged from the values predicted theoretically. Theory seems to under-predict the convective heat transfer coefficients for PMMA, which may be due to the same reasons the local flame stand-off and temperature gradients diverge from the LBL theory, described earlier. However, the crude approximation using Eq. (12) slightly over-predicts the heat transfer coefficient. The average value of the convective heat transfer coefficient was found to be 10.3 and 9.2 W/m²K for methanol and ethanol, respectively. For PMMA, the average value of the convective heat transfer coefficient was estimated to be 12.6 W/m²K.
Figure 12: Variation of convective heat transfer coefficients in the pyrolysis zone for a methanol, ethanol and PMMA boundary-layer diffusion flame.

5.7. Nusselt number

The Nusselt number was also calculated for methanol, ethanol and PMMA laminar boundary-layer diffusion flames. This parameter is equal to the dimensionless temperature gradient at the surface and provides a measure of the convective heat transfer occurring at the surface. The Nusselt number can be expressed as [41]

\[ \text{Nu} = \frac{h L}{k_w} = \frac{\partial T^*}{\partial y^*} \bigg|_{y^*=0} \]  \hspace{1cm} (15)

From the initial definition of \( T^* \), it follows that for a prescribed geometry,

\[ \text{Nu} = f(x^*, \text{Gr}, \text{Pr}) \]  \hspace{1cm} (16)

The Nusselt number is to the thermal boundary layer what the friction coefficient is to the velocity boundary layer. Eqn. (16) implies that for a given geometry, the Nusselt number must be some universal function of \( x^* \), \( \text{Gr} \) and \( \text{Pr} \). If this function were known, it could be used to compute the value of Nu for different fluids and for different values of \( L \) and \( U \) (for forced flow).

From knowledge of Nu, the local convection coefficient may be found and the local heat flux may then be computed using Eqn (11). Figure 13 shows the variation of Nu with non-dimensional distance downstream of the leading edge, \( x^* \) for a methanol, ethanol and PMMA wall flame. Theoretical results
based on the thermal boundary layer approximation are also plotted for comparison [42]. Nu calculated by using Eqn. (15) versus Nu calculated by using the thermal boundary-layer approximation are very similar and provide good agreement for both methanol and ethanol. However, for PMMA the Nusselt number calculated from theory under predicts the measured value. This is expected because vertical burning of PMMA was shown to not follow the LBL scaling rule. Correspondingly, the thermal boundary-layer approximation will not predict the Nu variation correctly.

Figure 13: Variation of Nusselt number in the pyrolysis zone for a methanol, ethanol and PMMA boundary-layer diffusion flame.

5.8. Plume Thermal Analysis

The fire and thermal plume plays an important role in flame spread, pre-heating unburnt fuel ahead of the flame front. The wall plume is also of fundamental interest as it differs from unconfined plumes, combining plume behavior and wall boundary-layer effects. The plume in this paper is defined as the region above (downstream) the pyrolysis front. Detailed temperature measurements in a wall plume were carried out for methanol, ethanol and PMMA boundary-layer diffusion flames at several stream-wise locations downstream of the pyrolysis zone. The temperature profiles in the combusting and thermal plume were mapped at 7 steamwise locations, shown in Figure 14. The temperature measurements were compensated for radiation losses (see appendix) and represent the averages of at least five independent tests.
The maximum temperature in all cases decreases monotonically with increasing height along the wall. The reduction is not large in the pyrolysis zone, but the rate increases rapidly in the plume. The position of maximum temperature moves closer to the wall, in terms of $y/x$, as $x$ increases. It is to be noted that peak temperatures in the thermal plume decrease significantly when compared to the combusting plume region. The plume grows in size (length) and thickness moving downstream. The temperature gradients in the combusting plume were found to be relatively constant ($\sim 450$ K/mm) in both the methanol and ethanol wall-bounded flames. This behavior was typical for both flames until the tip of the flame was reached. Thereafter, temperature gradients decreased significantly downstream of the combusting plume. Intermittent heating of the wall surface by direct flame contact in the combusting plume region caused temperature gradients to increase at the wall, thereby contributing to higher incident fluxes (when compared to heat...
fluxes at the trailing edge of the condensed fuel surface) in the combusting plume region. The flame also comes back to the wall in the overfire region, heating up the wall surface to sufficiently high temperatures (on the order of 650 K in the combusting plume region). Thereafter, the wall temperature decreases significantly in the thermal plume region. Figure 15 shows the temperature vs. time data in the combusting plume of a methanol diffusion flame at three different y-locations for $x/L = 1.31$. Temperature fluctuations were found to be high on the air and fuel side of the flame and were found to be low at the location of the flame.

Figure 15: Variation of temperature with time in the combusting plume of a methanol boundary-layer diffusion flame.
5.9. Wall Heat Flux in the Plume or Overfire Region

The wall heat flux in the plume was measured at several stream-wise locations by a total heat flux gauge, a Medtherm model 64-10-20 mounted flush with the wall. The total heat flux measured by the heat flux gauge was de-convoluted into its convective and radiative components, shown in Figure 16. The convective flux was calculated by using the expression \( k_w \frac{dT}{dy} \bigg|_{y=0} \), where \( k_w \) was evaluated at the average plume-wall temperature. Also shown are heat fluxes resulting from a crude approximation given by Eqn. (11). The only difference in the respective formula was to use the value of \( T_{hfg,s} \) (\( \sim 286 \) K) in place of \( T_w \). It is to be noted that the surface of the heat flux gauge was kept at a constant temperature of 286 K using water-cooling and \( k_f \) was evaluated at the mean film temperature. Since the plume is characterized by large temperature variations, selection of transport properties at an appropriate temperature is essential. The transport properties were evaluated at a mean film temperature in the plume which was given by

\[
T_f = \frac{T_{max} + T_\infty}{2}.
\]

(17)

The selection of a mean film temperature is based on the work of Ahmad [43], where results, when correlated in this manner, agreed reasonably well with their theoretical model. The mean film temperature using this method was estimated to be \( \sim 1032 \) K for methanol and \( \sim 1007 \) K for ethanol.

No heating effects of insulation were observed on readings from the total heat flux gauge. Radial heat transfer was also found to be negligible. Figure 16 shows a summary of the heat flux data obtained at various sensor positions in the plume. Values are provided for the total heat flux and its radiative and convective components. The flame also radiates heat flux to the ambient which was found to be generally larger than the radiant flux reaching the wall [43]. This behavior has also been observed by Orloff et al. [20]. They attributed the effect to the absorption of radiation by the fuel and product-rich region near the wall. However, no attempt was made in this study to measure the radiant flux to the ambient, but these measurements can be found in literature elsewhere [44].
Figure 16: Plume heat fluxes at several stream-wise locations along the wall for (left) methanol and (right) ethanol boundary layer diffusion flames.

The heat flux to the wall increases just beyond the pyrolysis zone as the effect of blowing decays. However, the heat flux remains relatively constant throughout much of the combusting plume region. Therefore, the presence of the flame in the combusting plume provides an extended zone of high wall heat flux which heats the combustible. It was observed that the heat flux increases from the trailing edge of the condensed fuel surface to a maximum and then decreases rapidly. The heat flux is low in the pyrolysis zone (towards the trailing edge of the condensed fuel surface), due to the blowing effect of fuel evaporating at the wall. In the overfire region, the heat flux increases, within one pyrolysis zone length, to a relatively constant value which is maintained until the tip of the flame is approached. Beyond the tip of the flame, the heat flux decreases rapidly once again.

The total heat flux reaching the HFG (Heat Flux Gauge) in the plume was estimated to be 16 kW/m² and 18.6 kW/m² for methanol and ethanol flames respectively. The average convective heat flux for a methanol and ethanol flame was estimated to be 13.8 kW/m² and 15.6 kW/m², respectively in the overfire region. The average radiative heat flux was then estimated to be 2.2 kW/m² and 3 kW/m² for methanol and ethanol flames respectively. The radiative heat flux to the wall was estimated to be 13.9 % and 16 % of the total wall heat flux for methanol and ethanol flames, respectively. It was observed that even in the plume region, the convective heat flux was the dominant mode of heat transfer to the wall. Even in the plume, radiation never accounts for more than 20% of the total wall heat flux for the present tests. The present low radiation levels are partly due to the fact that the
fuels tested were relatively soot-free. Longer pyrolysis zone lengths would increase the contribution of radiation to the wall heat flux. Far above the fire in the thermal plume region, the radiant contribution is small and heat transfer rates are dominated by convection.

Although there have been numerous studies on boundary-layer diffusion flames established over condensed fuel surfaces, only Ahmad and Faeth [29] have evaluated the various components of wall heat flux in the laminar and turbulent fire plume with smaller burning samples. They estimated the radiative heat flux contribution to be small, never accounting for more than 20% of the total flame heat flux in both the laminar and turbulent fire plumes. This is in accordance with our observations of these laminar fire plumes.

6. Conclusion

The knowledge of local mass burning rates and temperature distributions along a condensed fuel surfaces were utilized to deconvolute the convective and radiative components of total flame heat flux in the pyrolysis region. Additionally, a total heat flux gauge was used in the plume together with high spatial resolution temperature measurements to deconvolute the convective and radiative components of total flame heat flux incident on the heat flux gauge. The methodology used here is novel in the sense that it allows determination of the various components of flame heat flux accurately both in the pyrolysis and plume regions by knowledge of local temperature gradients along the condensed fuel surfaces. Also, the work presented here discusses the selection of transport properties at appropriate temperatures that allows researchers to calculate convective fluxes by using a crude approximation.

7. Acknowledgments

The authors would like to acknowledge financial support for this work from the Minta Martin Foundation at the University of Maryland, College Park.

Appendix A. Radiation Correction for the Thermocouple

The temperature measurements reported in this study have been corrected for thermocouple radiation. Flame temperature measurements across the width of the fuel sample showed no significant variation, except near the
edges. Therefore, thermocouples at the center of the flame were used to produce a map of temperatures in the boundary layer by moving it across the flame ($y$-direction) and along the length of the flame ($x$-direction). Two thermocouples (50µm and 75µm wire-diameter) were traversed along the same path at the center of the flame for accurate radiation corrections. All temperature measurements reported in this paper are an average of at least five independent tests conducted under the same conditions. In the most general case, an energy balance on the thermocouple junction takes the following form,

$$\dot{Q}_{\text{cat}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} + \dot{Q}_{\text{cond}} = m_{tc}c_p \frac{dT_{tc}}{dt}. \quad (A.1)$$

with heat transfer associated with surface-induced catalytic reactions, convection between the gases and the thermocouple, radiant heat transfer between the thermocouple and its surroundings, conduction along the thermocouple wires, and transient heating or cooling of the thermocouple incorporated in Eq. (A.1). The thermocouple junction properties that characterize the transient term in the above expression include the mass of the thermocouple junction, $m_{tc}$ and the specific heat $c_p$. For transient measurements, the convection and thermal inertia terms are both important, in addition to radiation. Neglecting the conduction error and errors due to catalytic effects Eq. (A.1) reduces to the following form for transient measurements,

$$T_g - T_{tc} = \frac{m_{tc}c_p}{hA_{tc}} \frac{dT_{tc}}{dt} + \frac{\varepsilon_{tc}\sigma}{h} (T_{tc}^4 - T_{surr}^4) \quad (A.2)$$

$$\tau \frac{dT_{tc}}{dt} + \frac{\varepsilon_{tc}\sigma}{h} (T_{tc}^4 - T_{surr}^4) \quad (A.3)$$

where $\tau$ is the characteristic response time or time constant of the thermocouple. Eq. (A.3) shows that the time constant of the thermocouple is not only related to the physical properties of the thermocouple i.e. the mass of the thermocouple junction, $m_{tc}$, the specific heat, $c_p$, and the surface area of the junction, $A_{tc}$, but also depend on heat transfer coefficient of the flow, $h$. There is a substantial literature devoted to the measurement of time constant of the thermocouple [45, 46, 47, 48, 49, 50].

For steady state measurements, as in our case, Eq.(A.1) reduces to a convective-radiative heat balance (neglecting the conduction error and errors due to catalytic effects) given by,
\( h (T_g - T_{tc}) = \varepsilon_{tc} \sigma \left( T_{tc}^4 - T_{surr}^4 \right) \)  \hspace{1cm} (A.4)

\( (T_g - T_{tc}) = \frac{\varepsilon_{tc} d_w \sigma}{k Nu} \left( T_{tc}^4 - T_{surr}^4 \right). \)  \hspace{1cm} (A.5)

where \( T_g \) is the real gas temperature, \( T_{tc} \) is the thermocouple junction (or bead) temperature, \( T_{surr} \) is the temperature of the surroundings, \( \varepsilon_{tc} \) is the emissivity of the thermocouple junction, \( \sigma \) is the Stefan-Boltzmann constant and \( h \) is the convective heat transfer coefficient of the flow over the thermocouple junction defined as \( h = k \frac{Nu}{d_w} \). \( k \) is the thermal conductivity of the gas, \( Nu \) is the Nusselt number, and \( d_w \) is the thermocouple wire diameter.

The choice of the Nusselt number correlation is of paramount importance in calculating a radiation correction to the measured thermocouple temperature because, as shown in Eq. (A.5), the radiation correction is inversely proportional to the Nusselt number. This choice is complicated, however, due to the existence of multiple “appropriate” Nusselt number correlations and the difficulty in estimation of the properties of the gas mixture surrounding the thermocouple, particularly its thermal conductivity. The bulk of evidence in literature, however, clearly indicates that a cylindrical Nusselt number correlation is most appropriate for describing the convective heat transfer to nearly all practical thermocouples [51], preferably that of Collis and Williams [34]. A commonly-used expression from Collis and Williams can be written as [34]

\[
Nu \left( \frac{T_m}{T_g} \right)^{-0.17} = 0.24 + 0.56 Re^{0.45} = 0.24 + 0.56 \left( \frac{U d_w}{\nu} \right)^{0.45}, \quad (A.6)
\]

which was obtained for \( 0.02 < Re < 44 \), with the Reynolds number evaluated at the so-called film temperature, \( T_m \) the mean of the thermocouple and free-stream temperatures, i.e. \( 0.5 (T_g + T_{tc}) \). Here, \( Re \) is the Reynolds number defined as indicated for the local gas flow velocity, \( U \) and kinematic viscosity, \( \nu \). Substituting Eq. (A.6) into Eq. (A.5) and neglecting the small temperature dependence in Eq. (A.6), we have a system of two equations with two unknowns (namely \( T_g \) and \( U \)),

\[
T_g - T_{tc1} = \frac{\varepsilon_{tc1} d_w1 \sigma}{k \left[ 0.24 + 0.56 \left( \frac{U d_{w1}}{\nu} \right)^{0.45} \right]} \left( T_{tc1}^4 - T_{surr}^4 \right) \quad (A.7)
\]
and
\[ T_g - T_{tc2} = \frac{\varepsilon_{tc2} d_{w2} \sigma}{k \left[ 0.24 + 0.56 \left( U d_{w2}/\nu \right)^{0.45} \right]} \left( T_{tc2}^4 - T_{surr}^4 \right), \] (A.8)

which demonstrates that the difference between a thermocouple reading and the actual gas temperature (i.e., the error in gas temperature measurement) increases for larger-diameter thermocouples, while it is reduced by increasing the gas flow velocity over the junction. In solving the above equations, iteration is required since the gas conductivity and kinematic viscosity are a function of the gas temperature. Initially, the gas temperature is taken to be the bead temperature for the purpose of evaluating the thermal conductivity and kinematic viscosity; then, the approximate value of the gas temperature is used to re-evaluate the thermal conductivity and viscosity.

The emissivity of the bead (\( \varepsilon_{tc} \)) can also be found as a function of its temperature. In an analysis outlined by Jakob [52], Maxwell's wave equations can be solved to yield the complex indices of refraction for a metal as a function of its electrical resistivity. In the limit of low resistivity and assuming a large index of refraction, which is true for metals, Jakob [52] gives the hemispherical total emissivity of platinum (Pt) as,
\[ \varepsilon = 0.751 (r_e T)^{1/2} - 0.396 (r_e T), 0 < r_e T < 0.2 \] (A.9)

where, for platinum, \( r_e = r_{e,273} T/273 \), with \( T \) in K and \( r_{e,273} = 11 \times 10^{-6} \) Ω-cm [53]. Therefore, the platinum emissivity becomes,
\[ \varepsilon = 1.507 \times 10^{-4} T - 1.596 \times 10^{-8} T^2 \] (A.10)

for \( 0 < T < 2230 \) K. This equation is also confirmed by comparison with experimental data [54]. It was shown [54] that for temperatures where radiation is important, predicted and observed emissivities agree to within 1%. The emissivity of the thermocouple bead or junction can therefore be evaluated by using the above expression. Note that iteration is not needed for the evaluation of the platinum emissivity, since this property is a function of the junction or bead temperature, which is known.

The actual gas temperature can then be evaluated by solving Eq. (A.7) and (A.8). During experiments, the two thermocouples were traversed exactly to the same measurement points and data was sampled to account for radiation correction in the temperature measurements.
Appendix B. Conduction Correction for the Thermocouple

Rapid thermal conduction along thermocouple wires can result in significant heat loss from the thermocouple wire and junction to the larger, cooler lead wires or support (cooler on account of increased radiation and conductive losses through the thermocouple support structure). However, this mechanism of heat loss from the thermocouple is usually avoidable through the use of sufficiently long and thin thermocouple wires on both sides of the junction. According to Bradley and co-authors [55] the conduction heat loss is assumed to be negligible if \( l > 200d_w \), where \( l \) is the length of the fine wire. However, a more detailed analysis by Petit and co-authors [56] reveals that a better criterion is to use wires of length \( l \) such that \( l/l_c > 10 \), in which \( l_c \) is the characteristic length defined as,

\[
l_c = \sqrt{\frac{k_w d_w}{4h_{conv}}}. \tag{B.1}
\]

This criterion accounts for both the characteristics of the flow and of the sensor. Values obtained from applying Petit criterion to the thermocouples used in this study with the exposed wire of 10 mm length and wire diameter of 50 \( \mu \)m at different locations in and out of the flame, were found to be in the range of 12-17 for the \( l/l_c \) ratio which is above the recommended value of 10. Overall, the conduction error is considered negligible in this study.

Appendix C. Theoretical Predictions of Convective Heat Transfer Coefficients

The expression for the boundary layer thickness for a free convection heat transfer on a vertical flat plate is given by [42],

\[
\frac{\delta}{x} = 3.93Pr^{-1/2}(0.952 + Pr)^{1/4}Gr_x^{-1/4} \tag{C.1}
\]

where \( Pr \) is the Prandtl number and \( Gr_x \) is the Grashof number. Convective heat transfer coefficient can then be derived as [42],

\[
h = \frac{2k}{\delta} \tag{C.2}
\]

The physical and transport properties are evaluated at the mean film temperature, \( T_f = (T_w + T_\infty)/2 \). For a boundary layer diffusion flame, this
might be approximated as $T_f = (T_w + T_{fl})/2$ where $T_{fl}$ is the average flame temperature for the given fuel and $T_w$ is the average wall temperature (taken as the pyrolysis temperature of the given fuel). Similarly, Grashof number takes the form,

$$Gr_x = \frac{g \beta (T_{fl} - T_w)x^3}{\nu^2}, \beta = \frac{1}{T_f} \quad (C.3)$$

Using the above equations, convective heat transfer coefficient can be calculated theoretically. Based on the convective heat transfer coefficient $Nu$ can be calculated correspondingly.

Appendix D. References


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